Handout 9 - Solutions

Aufgabe 1

- Erwarteter Payoff von 0:

$$Ui(0) = 0$$

- Erwarteter Payoff von 1= Erwarteter payoff von 0:

$$Ui(1) = p_0 99 + (1 - p_0)(-1) = 0$$

$$\Rightarrow p_0 99 + p_0 - 1 = 0$$

$$\Leftrightarrow p_0 = \frac{1}{100}$$

- Erwarteter Payoff von 2= Erwarteter payoff von 1:

$$Ui(2) = (p_0 + p_1)98 + (1 - p_0 - p_1)(-2) = 0$$

$$\Rightarrow p_0 98 + 98p_1 + 2p_0 - 2 = 0$$

$$\Leftrightarrow p_1 = \frac{1}{100}$$

Aufgabe 2

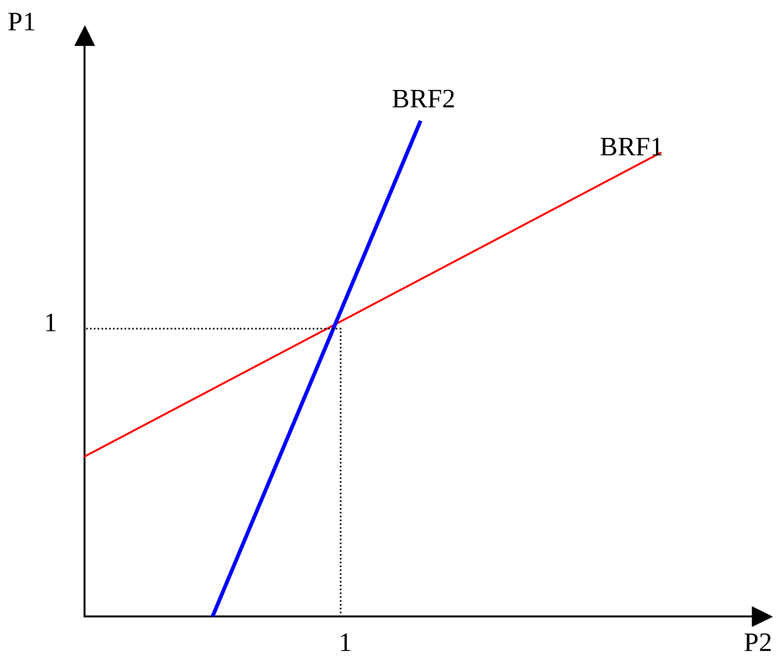
$$D_i = 1 - p_i + p_j$$

$$\Leftrightarrow \Pi_i = [1 - p_i + p_j]p_i$$

$$\begin{cases} \dfrac{\partial \Pi_i}{\partial p_i} = 1 - 2p_i + p_j \overset{!}{=} 0 \\ \Leftrightarrow p_i = \dfrac{1 + p_j}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} p_1 = \dfrac{1 + p_2}{2} \\ p_2 = \dfrac{1 + p_1}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} p^n{}_1 = 1 \\ p^n{}_2 = 1 \\ \Pi_i = 1 \qquad i = 1,2 \end{cases}$$



Aufgabe 3

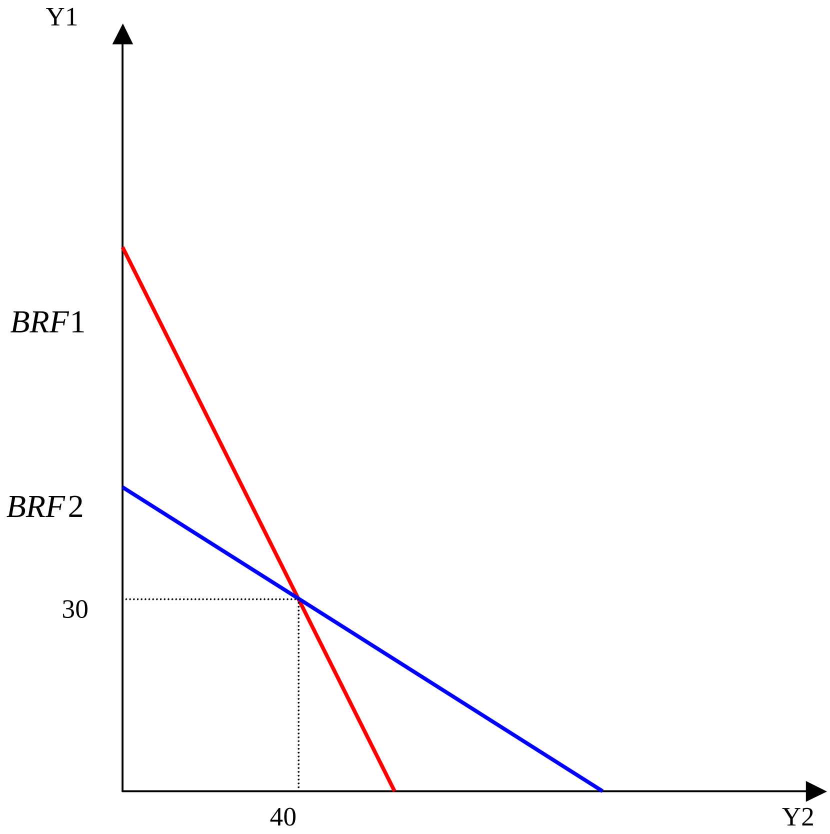
$$\Pi_i = py_i - C_i(y_i)$$

$$\Rightarrow \begin{cases} \Pi_1 = [120 - y_1 - y_2]y_1 - 20y_1 \\ \Pi_2 = [120 - y_1 - y_2]y_2 - 10y_1 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{\partial \Pi_1}{\partial y_1} = 100 - 2y_1 - y_2 \overset{!}{=} 0 \\ \Leftrightarrow y_1 = 50 - \dfrac{1}{2} y_2 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{\partial \Pi_2}{\partial y_2} = 110 - 2y_2 - y_1 \overset{!}{=} 0 \\ \Leftrightarrow y_1 = 55 - \dfrac{1}{2} y_1 \end{cases}$$

$$\Rightarrow \begin{cases} y^n{}_1 = 30 \\ y^n{}_2 = 40 \\ p^n = 50 \\ \Pi_1 = 900 \\ \Pi_2 = 1600 \end{cases}$$



Aufgabe 4

a) Die Nachfrage funktionnen einer konsumente der indiffernent zwischen A und B ist, sind:

$$p_1 + t(X - a)^2 = p_2 + t(1 - X)^2$$

$$\Rightarrow D_1(p_1, p_2, a) = X = \frac{p_2 - p_1}{2t(1-a)} + \frac{1+a}{2}$$

$$\Leftrightarrow D_2(p_1, p_2, a) = 1 - X = \frac{p_1 - p_2}{2t(1-a)} + \frac{1+a}{2}$$

b) Beste Antwort Funktionnen und Gleichgewicht:

$$\prod_1 (p_1, p_2, a) = (p_1 - c)\frac{p_2 - p_1 + t(1-a^2)}{2t(1-a)}$$

$$\Rightarrow \begin{cases} \dfrac{\partial \prod_1}{\partial p_1} = p_2 + c - 2p_1 + t(1-a^2) \overset{!}{=} 0 \\ \Leftrightarrow\ p_1 = \dfrac{c + t(1-a^2)}{2} + \dfrac{1}{2} p_2 \end{cases}$$

$$\prod_2 (p_1, p_2, a) = (p_2 - c)\frac{p_1 - p_2 + t(1-a)^2}{2t(1-a)}$$

$$\Rightarrow \begin{cases} \dfrac{\partial \prod_2}{\partial p_2} = p_1 + c + t(1-a)^2 - 2p_2 \overset{!}{=} 0 \\ \Leftrightarrow\ p_2 = \dfrac{c + t(1-a)^2}{2} + \dfrac{1}{2} p_1 \end{cases}$$

$$\Rightarrow \begin{cases} p^n{}_1 = c + t(1-a)\left(1 + \dfrac{a}{3}\right) \\ p^n{}_2 = c + t(1-a)\left(1 - \dfrac{a}{3}\right) \end{cases}$$

c)

$$\prod{}^n{}_1 = t(1-a)\left(1 + \frac{a}{3}\right)^2$$

$$\Rightarrow \frac{\partial \prod{}^n{}_1}{\partial a} = \left(1 + \frac{a}{3}\right)^2 \left(-\frac{t}{2}\right) + t(1-a)\left(1 + \frac{a}{3}\right)\frac{1}{3}$$

$$\Rightarrow \frac{\partial \prod{}^n{}_1}{\partial a} = t\left(1 + \frac{a}{3}\right)\left(-\frac{1+3a}{b}\right) < 0$$